Multifractal Fourier spectra and power-law decay of correlations in random substitution sequences

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Binary symbolic sequences produced by randomly alternating substitution rules are considered. Exact expressions for the characteristics of autocorrelations and power spectra are derived. The decay of autocorrelation function obeys the power law. The Fourier spectral measure is either absolutely continuous or a mixture of the absolutely continuous and singular continuous components. For the latter case, the multifractal characteristics of this measure are computed.

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Discrete patterns (vortices, crystals, spins, nucleotides, etc.) occur on a continuous background in almost every natural system. This makes such systems amenable to discretized description, the ultimate step of which is the reduction to a finite set of symbols, without restricting the generality [1]. For example, in nonlinear dynamics a common practice is to define the partition of the state space, assign labels to partition cells, and mimic continuous evolution by a sequence of letters. In many important cases the resulting symbolic code is invariant under replacement of certain blocks of several letters by one letter. Such self-similar codes are recovered, e.g., at the onset of chaos through the period-doubling scenario [2,3], quasiperiodicity [4], or homoclinic bifurcations [5]. In a wider context, self-similar symbolic sequences stand behind many phenomena in physics of critical states. Often exact similarity is marred by imperfections caused by various reasons, from the action noise to finite-size effects and influence of the boundaries, which are not necessarily compatible with self-similar patterns.

Inversely, symbolic codes are unfolded from a single initial symbol by repeated substitutions (inflations), which replace each letter by the prescribed block. Viewed as dynamical systems, deterministic substitution sequences have long been an object of intensive studies [6,1]. On assigning numerical values to letters, a symbolic string turns into a time series and can be characterized through entropies, Fourier spectra, correlation functions, etc. For periodic and quasiperiodic sequences the power spectrum is a set of discrete δ peaks; spectra of chaotic sequences are absolutely continuous with respect to the Lebesgue measure. One of the classical substitution sequences, the Thue-Morse code [7,8] produced by the action of a substitution $\{{A\to AB \atop B\to BA}$ on a two-letter alphabet, is neither periodic nor chaotic; accordingly, its power spectrum is neither discrete nor absolutely continuous, but was proved to be singular continuous [9]: the spectral measure is supported by the fractal set.

In this work, we introduce randomness into the substitution process: for each symbol, the substitution pattern is chosen among several candidates. We show how this leads to the power-law decay of correlations; for a class of such sequences, the Fourier spectrum is a multifractal mixture of absolutely continuous and singular parts.

We restrict ourselves to substitutions in which the alphabet is binary, all letters are updated simultaneously, and each one is replaced by two letters, so that one global update doubles the length of the symbolic string. The process starts with one letter and creates the infinite sequence $\{\xi_j\}$. In the biological context, this can be viewed as a toy model of linear growth: a cell divides into two, which divide again and so on. If there are only two kinds of cells, the process is a sequence of binary substitutions. Without mutations, all cells obey the same ''built-in'' division rule; the simplest example is a duplication $A\to AA$. With random mutations allowed, some of the divisions follow different rules, introducing local disorder into the growing pattern.

To conclude on the nature of spectral measure, we use the autocorrelation $C(\tau)=(\langle\xi_j\xi_{j+\tau}\rangle-\langle\xi_j\rangle^2)/(\langle\xi_j^2\rangle-\langle\xi_j\rangle^2)$ where average is taken over the position j in the string. In systems with discrete Fourier spectra, $C(\tau)$ displays repetitive peaks: if dynamics is periodic, peaks are located at the multiples of period and have unit height; in case of quasiperiodicity, the highest peaks correspond to rational approximations to the rotation number, and their height tends to 1. If the spectrum is absolutely continuous, $C(\tau)$ decays. Autocorrelation of the Thue-Morse sequence has a typical pattern of a system with purely singular power spectra [Fig. 1(a)] it is built around a log-periodic lattice of moderate peaks $C(3\times 2^n)=-C(2^n)=1/3$. Another tool is the integrated autocorrelation $C_{\rm int}(t)\equiv t^{-1}\Sigma_{\tau=0}^{t}C^2(\tau)$; the decay rate of $C_{\rm int}(t)$ yields the correlation dimension D_2 of the spectral measure: for t large, $C_{\rm int}(t)\sim t^{-D_2}$ [10].

We start with two inflation rules that randomly alternate. At each individual place the replacing pattern is chosen among two candidates: with probability p(w.p.p) the symbol is duplicated and with probability $1-p$ the complementary symbol is written after it,

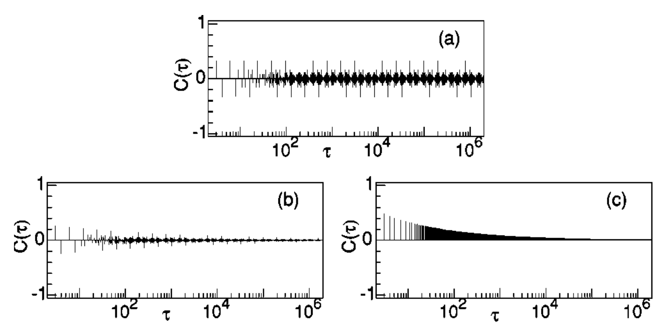

FIG. 1. Autocorrelation function for the substitution (1). (a) $p=0$ (Thue-Morse sequence), (b) $p=0.05$, (c) $p=0.9$.

$$A\rightarrow\begin{cases}AA & \text{w.p. } p\\ AB & \text{w.p. } 1-p,\end{cases}\qquad B\rightarrow\begin{cases}BB & \text{w.p. } p\\ BA & \text{w.p. } 1-p.\end{cases}\qquad (1)$$

The rule for each letter is chosen independently of the choices made elsewhere. Deterministic case $p=0$ yields the Thue-Morse code; the trivial opposite limit $p=1$ results in repetition of one symbol. For $p\neq 0,1$ the statistical treatment is required; statements below refer to the expectations of the corresponding values and assume averaging over ensembles of infinite symbolic strings.

Assigning numerical values to symbols (i.e., $A=-B=1$) and demanding invariance of the averaged products $\langle \xi_j \xi_{j+\tau}\rangle$ with respect to inflation (1), we obtain two recurrent relations for the autocorrelation function,

$$C(2\tau)=k_{\rm e}C(\tau),\qquad (2)$$

$$C(2\tau+1)=k_{\rm o}[C(\tau)+C(\tau+1)],$$

$$k_{\rm e}=1-2p+2p^2,\quad k_{\rm o}=p-\frac{1}{2},$$

which, together with the "initial condition"

$$C(1)=(2p-1)/(3+4p-4p^2),$$

determine $C(\tau)$ for any τ. For $p\neq 0,1$ the prefactor $k_{\rm e}$ in the first of Eqs. (2) lies between 0 and 1; hence, autocorrelation decays. The highest peaks obey the power law $|C(\tau)|\sim\tau^{\kappa}$ with $\kappa=\ln(1-2p+2p^2)/\ln 2$. For $p<1/2$ the second prefactor $k_{\rm o}$ is negative and the decay is oscillatory [Fig. 1(b)]; for $p>1/2$ the values of $C(\tau)$ stay positive [Fig. 1(c)].

The relations (2) allow us to determine the decay rate of the integrated autocorrelation $C_{\rm int}$. The evolution of the sums

$$U_n\equiv\sum_{\tau=2^n}^{2^{n+1}-1}C^2(\tau),\quad W_n\equiv\sum_{\tau=2^n}^{2^{n+1}-1}C(\tau)C(\tau+1)\qquad (3)$$

is governed by recurrent relations

$$U_{n+1}=(k_{\rm e}^2+2k_{\rm o}^2)U_n+2k_{\rm o}^2W_n+k_{\rm o}^2\zeta_n,$$

$$W_{n+1}=2k_{\rm e}k_{\rm o}(U_n+W_n)+k_{\rm e}k_{\rm o}\zeta_n,\qquad (4)$$

where $\zeta_n=[C(1)]^2(k_{\rm e}^2-1)k_{\rm e}^{2n}$. If U_n and W_n as functions of n do not decrease, the terms with ζ_n in Eqs. (4) can be neglected, and for large values of n both sums (3) are proportional to λ^n, where λ is the larger root of the quadratic equation

$$\lambda^2-\left(4p^4-4p^3+4p^2-2p+\frac{1}{2}\right)\lambda$$

$$-(1-2p)(2p^2-2p+1)^3=0.\qquad (5)$$

Accordingly, the decay of the integrated autocorrelation $C_{\rm int}(t)$ is described by the power law with the exponent $\max(\ln\lambda/\ln 2-1,-1)$. This yields the correlation dimension of the spectral measure,

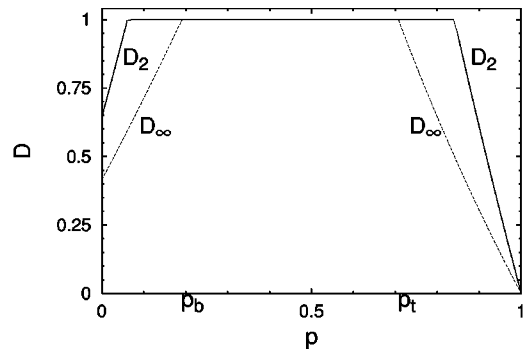

FIG. 2. Dependence of fractal dimensions of the spectral measure on probability p.

$$D_2=\min\left(1,1-\frac{\ln\lambda}{\ln 2}\right).\qquad (6)$$

Since for $p<0.061\,23$ and for $p>0.840\,79$, $D_2(p)$ is smaller than 1, the spectral measure in these intervals of p is fractal or, at least, includes a singular component (Fig. 2).

To get a better idea of the distribution of spectral measure as a function of p, consider dynamics of finite-length Fourier sums $S_n(\omega)=2^{-n}|\sum_{k=1}^{2^n}\xi_k\exp(i2\pi\omega k)|^2$ under the increase of n. Due to periodicity, analysis can be restricted to the interval $0\leqslant\omega<1$. For the absolutely continuous spectrum, $S_{n\to\infty}(\omega)$ converges to a bounded curve. Pointwise divergence of $S_n(\omega)$ indicates presence of singularities in the spectral measure. For the pure-point measure, $S_{n\to\infty}(\omega)$ vanishes everywhere outside the countable set of points; in these points, which correspond to δ peaks in the spectrum, S_n diverges and the ratio S_{n+1}/S_n tends to 2.

For the substitution (1), the Fourier sums are interrelated by the functional equation

$$S_{n+1}(\omega)=2p-2p^2+[1-2p+2p^2$$

$$-(1-2p)\cos 2\pi\omega]S_n(2\omega)\qquad (7)$$

with the initial condition $S_0=1$.

At $p=0$ this gives $S_{n+1}(\omega)=(1-\cos 2\pi\omega)\,S_n(2\omega)$. The sum $S_n(\omega)$ vanishes at $\omega=m/2^{n-1}$, $m=0,1,2,\ldots$. The values of ω with the highest local growth rate $\gamma_{\rm max}=S_{n+1}(\omega)/S_n(\omega)$ lie at $\omega=m/(3\times 2^{n-1})$; $m=1,\ldots,3\times 2^{n-1}-1$; since $\gamma_{\rm max}=1-\cos 2\pi/3=3/2<2$, the highest peaks grow slower than the δ peaks would do. Numerically, the multifractal spectral measure of the Thue-Morse sequence was analyzed in [11]; exact expressions for the generalized dimensions were derived in [12].

Under $p\neq 0,1$ the factor before $S_n(2\omega)$ in Eq. (7) is positive. Hence, $S_{n+1}(\omega)>2p-2p^2>0$, and the absolutely continuous part is present in the power spectrum.

The curve $S_n(\omega)$, typical for small p (here, $p=0.1$, $n=11$) is plotted in Fig. 3(a). For $p<1/2$, the rate $\gamma_{\rm max}$ equals $2p^2-3p+3/2$ and is attained at the same values of ω as for the Thue-Morse code; the highest peak for all n lies at $\omega=1/3$. Presence of the dense set of singularities is guaranteed if $\gamma_{\rm max}>1$; absence of the discrete component is ensured by $\gamma_{\rm max}<2$. The latter inequality holds for all p, and from the

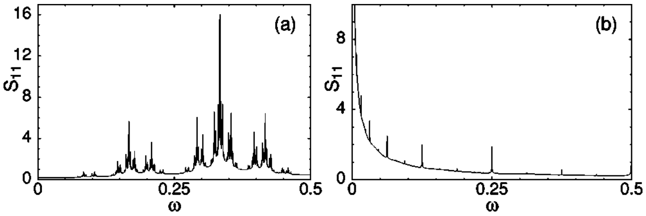

FIG. 3. Finite-length approximations $S_n(\omega)$ to spectral curves. (a) $p=0.1$, $n=11$; (b) $p=0.9$, $n=11$.

former it follows that for $p<p_b=(3-\sqrt{5})/2=0.1909\ldots$ the power spectrum is a mixture of absolutely continuous and singular continuous components. To the support of the latter belongs the disjoint continuum of ω values, which, written in the binary notation, contains the infinite number of sufficiently long segments $\cdots 101010101\cdots$.

When p exceeds 1/2, the picture is different [Fig. 3(b)]. Now $\gamma_{\max}$ equals $2p^2$ and is reached at $\omega=m/2^{n-1},m=0,1,2,\ldots$. For $p>p_t=\sqrt{2}/2=0.707\cdots$, the inequalities $1<\gamma_{\max}<2$ are fulfilled, and the power spectrum includes a dense set of singularities; the highest peak for each n lies at $\omega=0$. The background of the spectrum is formed by the continuous component, which obeys a power law: for the nonsingular small values of ω,

$$S_{n\to\infty}(\omega)\sim\omega^{-1-2\ln p/\ln 2}. \tag{8}$$

Accordingly, for $p_t<p<1$ the spectrum is again a mixture of absolutely continuous and singular components. The fractal support of the latter includes the continuum of ω values whose binary expansions contain infinitely many segments $\cdots 0000\cdots$.

For the values of p between p_b and p_t, the power spectrum contains no singularities; it is absolutely continuous. In the simplest case of $p=1/2$ when substitutions in Eq. (1) have equal probabilities, Eq. (7) results in $S_n(\omega)=1$: the spectral measure is uniform. According to Eq. (2), the autocorrelation $C(\tau)$ in this case vanishes identically.

For $0<p<p_b$ and for $p_t<p<1$ the distribution of the spectral measure is multifractal. The formalism of multifractal analysis [13,14] has been adapted for Fourier spectra in [11,15]: the range of values of ω between 0 and 1 is partitioned into the boxes of the size ε, and the partition function for a real q is introduced as $U(q,\varepsilon)=\Sigma_{i=1}^{1/\varepsilon}\rho_i^q$, where ρ_i is the probability to locate the measure in the ith box. Since spectral measure itself is not explicitly available, one can use as approximations the finite-length sums $S_n(\omega)$; as shown in [12], the refinement of the partition should be accompanied by the increase of n. Assuming the scaling $U(q,\varepsilon)\sim\varepsilon^{\tau(q)}$, we arrive in the usual way at the definition of generalized dimensions D_q:$D_q=(q-1)^{-1}\tau(q)$. Due to the presence of continuous background, $D_q=1$ for $q\leqslant 1$. For large positive q, the partition function is dominated by the contribution of boxes in which $S_n(\omega)$ grows with the rate $\gamma_{\max}$; accordingly,

$$D_{q\to\infty}\cong\frac{q}{q-1}(1-\ln\gamma_{\max}/\ln 2).$$

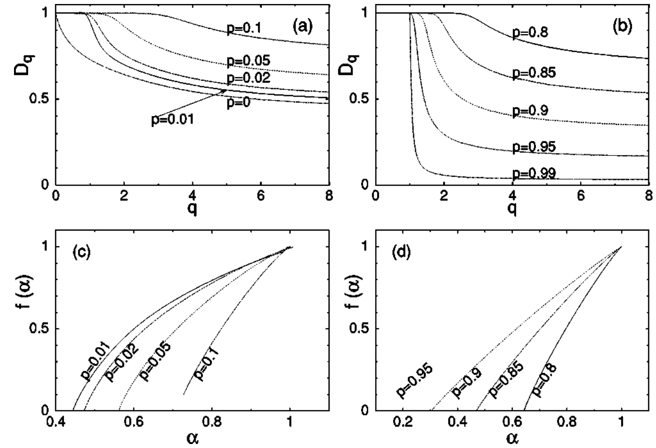

FIG. 4. Multifractal characteristics of the spectral measure of substitution (1). (a),(b) Generalized fractal dimensions; (c),(d) spectra of singularities.

For $0<p<p_b$, as shown above, $\gamma_{\max}=2p^2-3p+3/2$; hence $D_{q\to\infty}\to D_\infty=1-\ln(2p^2-3p+3/2)/\ln 2$. In the interval $p_t<p<1$, the rate $\gamma_{\max}$ equals $2p^2$; therefore, $D_\infty=-2\ln p/\ln 2$. Numerically found dependencies D_q for several values of p are plotted in Figs. 4(a) and 4(b).

The dependence $\tau(q)$ yields the spectrum of singularities $f(\alpha)$ [14]: $\alpha=d\tau/dq$ and $f(\alpha)=q\alpha-\tau$. The presence of the continuous component makes the distribution, in terms of [11], "half a multifractal": the curve $f(\alpha)$ has only an ascending branch [Figs. 4(c) and 4(d)]. For each p, the values of α lie in the range $(D_\infty,1)$ (in the exceptional case of purely singular continuous spectrum at $p=0$, the right border reaches $\alpha=2$ [11]). With increase of p from zero, this range monotonically decreases and at $p=p_b$ shrinks to a point; up from p_t the range of α regains the finite width.

Now consider random choice among all possible two-letter words. Fix nonnegative p_1, p_2, and p_3 so that $p_4\equiv 1-p_1-p_2-p_3\geqslant 0$, and inflate each symbol according to

$$A\to\begin{cases}AA & \text{w.p. } p_1\\ AB & \text{w.p. } p_2\\ BA & \text{w.p. } p_3\\ BB & \text{w.p. } p_4,\end{cases}\qquad B\to\begin{cases}BB & \text{w.p. } p_1\\ BA & \text{w.p. } p_2\\ AB & \text{w.p. } p_3\\ AA & \text{w.p. } p_4.\end{cases} \tag{9}$$

The values of the autocorrelation function $C(\tau)$ for the substitution (9) are related through the recurrence

$$C(2\tau)=K_{\rm e}C(\tau),\quad C(2\tau+1)=K_{\rm o}[C(\tau)+C(\tau+1)] \tag{10}$$

with $K_{\rm e}=(p_1-p_4)^2+(p_2-p_3)^2$ and $2K_{\rm o}=(p_1-p_4)^2-(p_2-p_3)^2$. This, again, ensures the power-law decay of autocorrelation: $|C(\tau)|\sim\tau^{\ln K_{\rm e}/\ln 2}$. The correlation dimension of the spectral measure is given by $D_2=\min(1,1-\ln\lambda/\ln 2)$, where λ is the larger root of

$$\lambda^2-(2K_{\rm o}^2+2K_{\rm e}K_{\rm o}+K_{\rm e}^2)\lambda+2K_{\rm e}^3K_{\rm o}=0. \tag{11}$$

In the case of "rare mutations" when one of the probabilities p_i is sufficiently close to 1, and the other three are small, the inequality $\lambda > 1$ (which, in turn, implies $D_2 < 1$) holds. Accordingly, the presence of the singular continuous component is typical for such situations.

Multifractal properties of the spectral measure can be recovered from finite-length sums $S_n(\omega)$ which, for the substitution (9), obey the recurrent relation

$$S_{n+1}(\omega)=1-K_{\mathrm{e}}+(K_{\mathrm{e}}+2K_{\mathrm{o}}\cos 2\pi\omega)S_n(2\omega)$$
$$+(p_1+p_4-p_2-p_3-2K_{\mathrm{o}})\cos 2\pi\omega. \quad (12)$$

If at least two of the probabilities p_i do not vanish, the spectral background is everywhere (except, perhaps, at $\omega=0,1/2$) bounded away from zero. Hence, the absolutely continuous component is present, and the power spectrum is either purely absolutely continuous or mixed.

The detailed analysis of the substitution (9) and the recurrent relation (12) will be presented elsewhere. The case $p_3=p_4=0$ has been considered above. Here we briefly comment on two other combinations of two substitutions. Under $p_1=p_4=0$ the alternating substitution rules are the Thue-Morse rule, and the "reordered Thue-Morse" rule. For each of them, taken alone, the Fourier spectrum is purely singular continuous. As soon as they are mixed together, the absolutely continuous component appears. In order for the spectrum to be multifractal, probabilities of the substitutions should strongly differ. If $p\equiv\min(p_2,p_3)$ exceeds 0.03, the dimension D_2 of the spectral measure equals 1. For $p>1/2-\sqrt{6}/6$, the dimension D_∞ turns into 1, and the power spectrum is absolutely continuous.

In the opposite case $p_2=p_3=0$ each of two competing rules produces the periodic symbolic sequence. For $p\equiv\min(p_1,p_4)<p_b=1/2-\sqrt{2}/2$, spectral measure has a singularity at $\omega=0$; at small ω the power law holds,

$$S_{n\to\infty}(\omega)\sim\omega^{-1-2\,\ln(1-2p)/\ln 2}. \quad (13)$$

Above p_b the singularity is absent; the spectrum is absolutely continuous.

Summarizing, random combination of binary substitutions always leads to the power-law decay of autocorrelation and is, in many cases, a cause of singularities in the spectral measure.

The power-law correlation decay ("long-range correlations") is abundant in natural processes, from physical systems near critical points [16] to human walking [17] and standing [18], atmospheric variability [19,20], and sequences of nucleotides in the DNA [21–25]. A decade ago Li recognized that many properties of such processes were reproduced by randomly alternating substitutions; in his "expansion-modification system," the choice was made between a duplication and a mutation to a complementary symbol [26,27]. Compared to the substitution (9), such systems are reminiscent of the case $p_2=p_3=0$: when mutations are rare, the power spectrum decays as $1/\omega^\alpha$ with $\alpha\sim 1$.

Processes with long-range correlations are often explained in terms of random walks [16–21], by means of squared fluctuation $F^2(l)=\langle(\Sigma_{j+1}^{j+l}\xi_i)^2\rangle$ averaged over the position j. In the scaling dependency $F^2(l)\sim l^{2\alpha}$, the value $\alpha=1/2$ is interpreted as an attribute of a genuine noncorrelated random walk, whereas $\alpha>1/2$ indicates the presence of correlations. Calculation for Eq. (9) yields $2\alpha=\max(\mathrm{sgn}[p_1+p_4],2+2\ln|p_1-p_4|/\ln 2)$. Thereby, the criterion does not distinguish substitutions with $|p_1-p_4|<\sqrt{2}/2$ from random walk; in particular, all sequences with a big proportion of Thue-Morse-like inflations (and slow correlation decay) look from this point of view as completely disordered.

Alternatively, in dynamics the degree of irregularity is usually expressed in terms of ergodic characteristics like mixing, which, in their turn, are related to spectral properties [1]. Analysis of observables built on two-letter words of inflations (1) and (9) shows that corresponding dynamical systems are neither mixing nor even weakly mixing. This means that, in spite of the continuous component in the Fourier spectrum, the actual level of disorder in the substitution sequences is relatively low.

Simple random substitution rules allow us to arrive at explicit scaling laws and exact values of fractal dimensions. Corresponding characteristics of more realistic models can be reasonably close to these laws and values. In this respect, it can be noticed that the strongly pronounced peak exactly at the frequency $\omega=1/3$ in Fig. 3(a) (as well as in the power spectra of the substitution (9) with dominating values of p_2 or p_3) is reminiscent of the "period-3" pattern recovered in the structure of correlation functions and mutual information of the DNA [28,29].

As seen from our analysis, power-law decay of correlations does not necessarily imply that the spectral measure is multifractal; however, in situations when one of the substitution patterns is strongly preferred, presence of fractal component is quite probable. To our knowledge, decomposition of spectral measure in processes with power-law correlations has not yet received the proper attention. The presented results allow to expect the fingerprints of fractal power spectra in many of such processes.

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